R7592

Sub. Code
511101

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

First Semester

Mathematics

GROUPS AND RINGS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions.

1. Let G be the group of nonzero real numbers under multiplication, and let H be the subset of positive rational numbers. Then H is a ———————— of G.
   (a) subgroup
   (b) simple group
   (c) symmetric subgroup
   (d) normal subgroup

2. A group G is said to be ———————— if for every $a,b \in G$, $a.b = b.a$
   (a) symmetric group (b) ring
   (c) abelian (d) non-abelian group

3. A subgroup N of G is said to be ———————— of G if for every $g \in G$ and $n \in N, gng^{-1} \in N$.
   (a) symmetric subgroup
   (b) simple subgroup
   (c) abelian subgroup
   (d) normal group

4. If G is finite group and N is a normal subgroup of G, then $o(G/N)=$

(a) $o(G)/o(N)$ (b) $o(N)/o(G)$

(c) $o(N/G)$ (d) $o(G)$

5. Every permutation is the product of its ————————

(a) indices (b) subgroups

(c) cycles (d) conjugates

6. If $a,b \in G$, then b is said to be a ———————— of a in G if there exists an element $c \in G$ such that $b = c^{-1}ac$.

(a) associate (b) transposition

(c) permutation (d) conjugate

7. A ring is said to be a ———————— if its nonzero elements form a group under multiplication.

(a) commutative ring (b) division ring

(c) field (d) integral domain

8. An integral domain D is said to be of ———————— if there exists a positive integer m such that $ma = 0$ for all $a \in D$.

(a) finite characteristic

(b) infinite characteristic

(c) countable characteristic

(d) uncountable characteristic

9. An integral domain R with unit element is a ———————— if every ideal A in R is of the form $A = (a)$ for some $a \in R$.

(a) Euclidean ring (b) maximal ideal

(c) Principal ideal ring (d) field

10. Let R be a commutative ring with unit element. An element $a \in R$ is a ———————— in R if there exists an element $b \in R$ such that $ab = 1$.

(a) greatest common divisor

(b) unit

(c) characteristic

(d) associate

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove any one equivalent criterion for a non-empty sub-set H of a group G is a subgroup of G.

Or

(b) Prove that there is a one-to-one correspondence between any two right cosets of H in G.

12. (a) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

Or

(b) Show that a homomorphism ϕ of G into G' with kernel K_ϕ is an isomorphism of G into G' if and only if $K_\phi = (e)$.

13. (a) Prove that every permutation can be uniquely expressed as a product of disjoint cycles.

Or

(b) Show that the normalizer of an element a in G is a subgroup of G.

14. (a) If ϕ is a homomorphism of R into R' with kernel $I(\phi)$, then show that $I(\phi)$ is a subgroup of R under addition.

Or

(b) If U is an ideal of the ring R, then show that R/U is a ring and is a homomorphic image of R.

15. (a) Prove that a Euclidean ring possesses a unit element.

Or

(b) Let R be a Euclidean ring. Suppose that for $a, b, c \in R$, $a \mid bc$ but $(a, b) = 1$. Then show that $a \mid c$.

Part C (5 × 8 = 40)

Answer any **five** questions.

16. Prove the following

(a) HK is a subgroup of G if and only if $HK = KH$.

(b) If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$, respectively, then $o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$.

17. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

18. State and prove the fundamental theorem of homomorphism of groups.

19. State and prove Cauchy theorem.

20. State and prove second part of Sylow's theorem.

21. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field.

22. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

23. Show that every integral domain can be imbedded in a field.

R7593

Sub. Code
511102

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

First Semester

Mathematics

REAL ANALYSIS — I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions.

1. If X is a metric space, if $E \subset X$ then the closure of E is

   (a) $\overline{E} = E \cap E'$ (b) $\overline{E} = E \cup E'$

   (c) $\overline{E} = (E \cap E') \cup E$ (d) $\overline{E} = (E \cup E') \cap E$.

2. Which of the following sets are compact

   (a) N (b) Q

   (c) $[a, b]$ (d) $[a, \infty]$

3. The series $\frac{1}{2^2+1} + \frac{\sqrt{2}}{3^2+1} + \frac{\sqrt{3}}{4^2+1} + \ldots$ is

   (a) Convergent (b) Divergent

   (c) Oscillators (d) None of these

4. The sequence $\lim_{n \to \infty} \left[3 + (-1)^n\right]$ is

   (a) Convergent (b) Divergent

   (c) Oscillatory (d) None of these

5. Find the radius of convergence for the power series $\sum_{v=1}^{\infty}\left[\frac{7^n}{n(3x-1)^{n-1}}\right]$

(a) 0 (b) $\frac{2x+1}{6}$

(c) $7|3x-1|$ (d) $5|x+1|$

6. The series $\sum_{x=0}^{\infty} x^n$ if $x \geq 1$ then the series

(a) Convergent (b) Diverges

(c) Oscillators (d) None of these

7. If (x, d) is a metric space and $A \subseteq X$ and if $d(x, A) = 0 \ \forall \ x \in x$, then

(a) A is closed (b) A is compact

(c) A is dense in X (d) None of these

8. Which of the following functions are uniformly continuous

(a) $f(x)=1/x$ in $(0,1)$ (b) $f(x)=x^2$ in $\mathbb{R}$

(c) $f(x)=\sin^2 x$ in $\mathbb{R}$ (d) $f(x)=\cos x$ in $\mathbb{R}$

9. The value of $\lim_{x\to 0}(x \sin 1/x)$ is

(a) ∞ (b) -1

(c) 0 (d) 1

10. Compute the value of $\lim_{x\to 5}\frac{x^3-125}{x^2-8x+15}$

(a) 0 (b) $-75/2$

(c) 75/4 (d) 75/2

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that every infinite subject of a countable set A is countable.

Or

(b) Prove that a set E is open if and only if its complement is closed.

12. (a) Suppose $\{s_n\}$ is monotonic then prove that $\{s_n\}$ converges if and only if it is bounded.

Or

(b) If Σa_n converges absolutely then prove that Σa_n converges.

13. (a) Prove that e is irrational.

Or

(b) If $\Sigma a_n = A$ and $\Sigma b_n = B$ then $\Sigma(a_n + b_n) = A + B$ and $\Sigma C a_n = CA$. for and fixed C.

14. (a) A mapping of a metric space X into a metric space Y is continuous on X iff if $f^{-1}(v)$ is open in X for every open set v in y.

Or

(b) Let E be a non compact set in $\mathbb{R}'$. Then P.T. there exists a continuous function on E which is not bounded.

15. (a) Let f be defined on $[a, b]$. If f has a local maximum at a P.T. $x \in (a, b)$ and $f'(x)$ exists then prove that $f'(x) \equiv 0$.

Or

(b) State and prove mean-value theorem.

Part C $(5 \times 8 = 40)$

Answer any **five** questions.

16. State and prove Weierstrass theorem.

17. State and prove ratio rest

18. P.T. $\lim_{n\to\infty} (1+a/n)^n = e$.

19. Suppose f is a continuous mapping of a compact metric space x into a metric space y then prove that $f(x)$ is compact.

20. State and prove Taylors theorem.

21. If $p > 0$ then $\lim_{n\to\infty} \sqrt[n]{P} = 1$.

22. Prove that it $P > 1 \sum_{n=2}^{\infty} \frac{1}{(n \log n) p}$ converges if $P \le 1$ then the series diverges.

23. State and prove L'hospital's rule.

R7594

Sub. Code
511103

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** questions.

1. If φ_1, φ_2 are two solutions of $\eta'' + a_1\varphi' + a_2\varphi = 0$ on an interval I containing a point x_0, then $w(\varphi_1, \varphi_2)(x) = Lw(\varphi_1, \varphi_2)(x_0)$ where L is ————.

   (a) $e^{a_1(x-x_0)}$ (b) $e^{a_2(x-x_0)}$

   (c) $e^{-a_1(x-x_0)}$ (d) $e^{-a_2(x-x_0)}$

2. Polynomials of degree two are linearly dependent on

   (a) $(-\infty, \infty)$ (b) $(-\infty, 0)$

   (c) $(0, \infty)$ (d) $(1, \infty)$

3. Given the n^{th} Legendre polynomial $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$ the co-efficient of x^n is

   (a) $\frac{n!}{2^n (n!)^2}$ (b) $\frac{2n!}{2^n (n!)^2}$

   (c) $\frac{2n!}{2^n n!}$ (d) $\frac{(2n!)^2}{2^n (n!)^2}$

4. Which of the following is called as Hermite equation,

   (a) $ay' - bxy' + bay = 0$ where $b = 2$

   (b) $a\,y'' + bxy' + bay = 0,$ where $a = 2$

   (c) $y'' - bxy' + bay = 0$, where $b = 2$

   (d) $y'' + bxy' + bay = 0,$ where $b = 2$

5. Find the singular point of the equation $a_0(x)\,y^n + a_1(x)\,y^{n-1} + \ldots + a_n(x)\,y = 0$.

   (a) $a_1(x_2) = 1$

   (b) $a_n(x_n) = \infty$

   (c) $a_0(x_0) = 0$

   (d) $a_0(x_0) = 1$

6. The singular point of the equation $x^2 y'' - y' - \frac{3}{4} y = 0$.

   (a) $x_0 = 0$ (b) $x_0 = -1$

   (c) $x_0 = -2$ (d) $x_0 = \frac{-3}{4}$

7. $f(x, y) = y^2$, f has derivatives of all orders with respect to x and y in (x, y)-plane. Find the general solution $\phi(x)$.

   (a) $\phi(x) = \frac{-1}{x}$

   (b) $\phi(x) = -1$

   (c) $\phi(2) = \frac{-2}{3}$

   (d) $\phi(x) = \frac{-y}{x}$

8. First order equation $y' = f(x, y)$ is said to have variable separated if and only if it can be written in the form

(a) $f(x, y) = \dfrac{g(x)}{h(y)}$

(b) $f(x, y) = g(y) + h(x)$

(c) $f(x, y) = g(x) - h(y)$

(d) $f(x, y) = \dfrac{h(y)}{g(x)}$

9. f, g are both functions on R. Suppose there exists positive constant ε and δ such that $\left|f(x, y) - g(x, y)\right| \le \varepsilon$ and $\left|y_1 - y_2\right| \le \delta$ then f, g are

(a) discontinuous function

(b) continuous function

(c) solution less function

(d) approximation function

10. The differential equation is $y' + y\tan X = \cos X$, $y(0) = 0$. The value of $y(\pi)$ is

(a) π (b) 2π

(c) -2π (d) $-\pi$

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find all solutions of the following equations,

(i) $y' + 4y = \sin 3x$

(ii) $y'' + y = 2\sin x$.

Or

(b) Compute the solution ϕ of $y''' - xy = 0$ which satisfies $\phi(0) = 1$, $\phi'(0) = 0$, $\phi''(0) = 0$.

12. (a) State and prove Existence theorem for analytic coefficients.

Or

(b) Find a solution ϕ of $y'' = -\frac{1}{2y^2}$ satisfying $\phi(0) = 1, \phi'(0) = -1$.

13. (a) Find the solution of the following equation for $x > 0$, $x^3 y''' + 2x^2 y'' - xy' + y = 0$.

Or

(b) Compute the first four successive approximations for $y' = 1 + xy$ and $y' = y^2$ for both the initial value is $y(0) = 1$.

14. (a) Consider the problem $y' = 1 - 2xy,\ y(0) = 0$.

(i) since the differential equation is linear, an expression can be found for the solution. Find it

(ii) show that approximation ϕ_3 satisfies $|\phi(x) - \phi_3(x)| < 0.01$ for $|x| \le \frac{1}{2}$.

Or

(b) Solve :

(i) $y'' + e^x y' = e^x$

(ii) $y^2 y'' = y'$.

15. (a) Prove that the series defining J_0 and K_0 converge for $|x| < \infty$.

Or

(b) Let ϕ_1, ϕ_2 be two solutions of $L(y) = 0$ on an interval *I*, and let x_0 be any point in *I*. Then ϕ_1, ϕ_2 are linearly independent on *I* if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$.

Part C (5 × 8 = 40)

Answer any **five** questions.

16. Suppose ϕ is a solution of $y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = 0$, $\varphi(x) = \phi(x) \exp\left(a_1 \frac{x}{n}\right)$. Show that φ satisfies a linear homogeneous equation with constant co-efficients.

$y^{(n)} + b_1 y^{(n-1)} + \ldots + b_n y = 0$, with $b_1 = 0$.

17. Let $\phi_1, \ldots, \phi_n$ be n solutions of $L(y) = 0$ on an interval *I* containing a point x_0. Then prove that $W(\phi_1, \phi_2, \ldots, \phi_n)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2, \ldots \phi_n)(x_0)$.

18. Let p be any polynomial of degree *n*, and let $p = c_0 p_0 + c_1 p_1 + \ldots + c_n p_n$, where $c_0, c_1, \ldots, c_n$ are constants. Show that

$$c_k = \frac{2k+1}{2} \int_{-1}^{1} p(x) p_k(x) \, dx, \ (k = 0, 1, \ldots, n).$$

19. Find the solution of the second order Euler equation $x^2 y'' + axy' + by = 0$, (*a*, *b* constants) and the polynomial q given by $q(r) = r(r-1) + ar + b$.

20. Find a solution ϕ of the form $\varphi(x) = x^r \sum_{k=0}^{\infty} c_k x^k$, $(x > 0)$ for the following equations;

(a) $2x^2 y'' + (x^2 - x)y' + y = 0$

(b) $x^2 y'' + (x - x^2)y' + y = 0$.

21. Let M, N be two real-valued functions which have continous first partial derivatives on some rectangle $R : |x - x_0| \leq a$, $|y - y_0| \leq b$. Then the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

22. Let f be a continous vector-value function defined on $R : |x - x_0| \leq a$, $|y - y_0| \leq b(a, b > 0)$ and suppose f satisfies a Lipschitz condition on R and K a Lipschitz constant for f in R_1 then prove that $|\phi(x) - \phi_k(x)| \leq \frac{M(K\alpha)^{K+1}}{K(K+1)!} e^{K\infty}$ for all X in I.

23. State and prove Local Existence theorem.

R7595

Sub. Code
511104

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

First Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 1 = 10)

Answer **all** the questions.

1. Every $n \geq 12$ is the sum of two ———————— numbers.
   (a) composite (b) prime
   (c) odd (d) even

2. If n is square, then $d(n)$ is ————————.
   (a) square (b) odd
   (c) even (d) 1

3. $\sum_{n>x} \frac{1}{n^2} = 0(x^{1-s})$ if ————————.
   (a) $s > 1$ and $x < 1$ (b) $s > 1$ and $x \geq 1$
   (c) $s < 1$ and $x \leq 1$ (d) $s < 1$ and $x \geq 1$

4. When do you say that two lattice points (a, b) and (m, n) are mutually visible?
   (a) $(a-m, b-n) = 1$ (b) $(a-b, m) = 1$
   (c) $(a-b, m-n) = 1$ (d) $(a, m-n) = 1$

5. Theorems relating different weighted averages of the same function are called ————————

(a) Abel's theorem

(b) Prime number theorem

(c) Tauberian theorems

(d) Merten's theorems

6. The prime number theorem implies

(a) $\lim_{x\to\infty}\frac{M(x)}{x}$ is undefined

(b) $\lim_{x\to 0}\frac{M(x)}{x}=1$

(c) $\lim_{x\to\infty}\frac{M(x)}{x}=0$

(d) None of these

7. If $a\equiv b\ (\text{mod}\, m)$ and $\alpha\equiv\beta(\text{mod}\, m)$, then we have

(a) $ax+\alpha\equiv bx+y(\text{mod}\, m)$, for all x and y

(b) $a\alpha\beta\equiv b(\text{mod}\ m)$

(c) $a^n\equiv b^n(\text{mod}\ m)$, for every positive integer n

(d) $f(a)-1\equiv(f(b))^2\ (\text{mod}\ m)$, for every polynomial f

8. $5n^3+7n^5\equiv$ ———————— for all integers n.

(a) 1 (mod 12) (b) $7(\text{mod}12)$

(c) $5(\text{mod}12)$ (d) $0(\text{mod}12)$

9. If P and Q are odd positive integers, then $(a^2n/p)=(n/p)$ whenever

(a) $(a,n)=1$ (b) $(a,\ p)=1$

(c) $(n,\ p)=1$ (d) None of these

10. What is the quadratic residue of an odd prime p if $p\equiv\pm1\ (\text{mod}10)$?

(a) 3 (b) 4

(c) 5 (d) 6

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove division algorithm.

Or

(b) If both g and $f * g$ are multiplicative, prove that f is multiplicative.

12. (a) Prove that for $x > 1$, $\sum_{n \le x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$, so the average order of $\phi(n)$ is $\frac{3n}{\pi^2}$.

Or

(b) State and prove Euler's summation formula.

13. (a) Prove that the following relations are logically equivalent

(i) $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$

(ii) $\lim_{x \to \infty} \frac{\vartheta(x)}{x} = 1$

(iii) $\lim_{x \to \infty} \frac{\psi(x)}{x} = 1$

Or

(b) Prove that every interval $[a, b]$ with $0 < a < b$ contains a rational number of the form p/q, where p and q are prime.

14. (a) Solve the congruence

(i) $5x \equiv 3 (\operatorname{mod} 24)$

(ii) $25x \equiv 15 (\operatorname{mod} 120)$.

Or

(b) State and prove Wolstenholm's theorem.

15. (a) Determine whether –104 is a quadratic residue or non-residue of the prime 997.

Or

(b) State and prove Euler's criterion.

Part C (5 × 8 = 40)

Answer any **five** questions.

16. State and prove fundamental theorem of arithmetic.

17. Prove that $\sum_{n \leq x} d(n) = x \log x + (2c-1)x + O(\sqrt{x}), \forall x \geq 1,$ where c is Euler's constant.

18. Prove that for $x \geq 2$, $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x + O(x),$ where the sum is extended over all primes $\leq x$.

19. Prove that for every integer $n \geq 2$, $\frac{1}{6} \frac{n}{\log n} < \pi(n) < 6 \frac{n}{\log n}$.

20. State and prove Lagrange's theorem.

21. Prove that the quadratic congruence $x^2 + 1 \equiv o (\text{mod } p)$, where p is an odd prime, has a solution if and only if $p \equiv 1 (\text{mod } 4)$.

22. Prove that the diaphantine equation $y^2 = x^3 + k$ has no solutions if k has the form $k = (4n-1)^3 - 4m^2$, where m and n are integers such that no prime $p \equiv -1 (\text{mod } 4)$ divides m.

23. State and prove quadratic reciprocity law.

R7596

Sub. Code
511505

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

First Semester

Mathematics

OBJECT ORIENTED PROGRAMMING AND C++

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   ($10 \times 1 = 10$)

Answer **all** questions.

1. Number of Object-Oriented Programming languages is ————.

   (a) 3   (b) 2

   (c) 8   (d) none

2. The extraction operator is ————.

   (a) <<   (b) >>

   (c) ::   (d) .*

3. The member of class is ————.

   (a) private   (b) public

   (c) both   (d) none

4. Which of the following is scope resolution operator?

   (a) ::   (b) .*

   (c) <<   (d) >>

5. Which pointer acts as an implicit argument to all the member functions?

(a) this (b) ptr

(c) new (d) none

6. How many virtual constructors in C++?

(a) 4 (b) 2

(c) 0 (d) 5

7. Number of types of polymorphism is ————.

(a) 3 (b) 2

(c) 4 (d) 8

8. Which one of the following operator in C++ can be overloaded?

(a) :: (b) .*

(c) . (d) =

9. The mechanism of deriving a new class from an old class is called ————.

(a) polymorphism

(b) inheritance

(c) constructors

(d) destructors

10. Which one of the following is third visibility modifier?

(a) protected (b) public

(c) private (d) none

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Write a brief note on *switch* statement and *break* statement with examples.

Or

(b) Discuss in detail of “cout” and “cin” with an example.

12. (a) Write a program to illustrate nesting of member functions in a class.

Or

(b) Explain inline functions with an example.

13. (a) Explain the relation between “pointers” and “arrays”.

Or

(b) Explain dynamic constructer with example.

14. (a) Explain function overloading.

Or

(b) Write a program to illustrate overloading operators.

15. (a) Write any six rules of virtual functions.

Or

(b) Write a program to illustrate hierarchical inheritance.

Part C (5 × 8 = 40)

Answer any **five** questions.

16. Write a brief note on Object Oriented Programming with applications.

17. Discuss in details of constructor and destructor with suitable examples.

18. Write a program that would sort a list of names in alphabetic order.

19. Explain the operators "new" and "delete" with example.

20. Explain overloading unary and binary operators with example.

21. Write a program which uses overload operators to convert from rectangle coordinate system to polar coordinate system.

22. Write a program to illustrate hybrid inheritance.

23. Differentiate between "Private inheritance" and "Public inheritance".